KINETIC DERIVATION OF TANGENT EQUATION.

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The following investigation is, I am afraid, more interesting than valuable, though it gives a very simple way of getting the equation of a tangent. But there is evidently an underlying principle which I have not succeeded in getting at, which would, if found, undoubtedly be valuable.

Since a curve can be considered as generated by the motion of a point, we can consider this point as the tracing point of a mechanism so adjusted as to trace the proper curve. This tracing point can be imagined to be the common point of various sets of x and y arms, each set composed of a certain number of x points, or of y points.

Now if, for example, the mechanism be set for the curve represented by the equation $ax^2+by^2=c$, and we stop the action of an x and a y point, leaving an x and a y point current, the new curve will be a straight line in *continuation* of the old motion. That is, the straight line will be a tangent line.

The algebraic equivalent of this would be found by writing

$$axx_1+byy_1=c$$

which would consequently be the *equation of the tangent line*, found by stopping one pair of coördinates (indicated by the subscripts) and letting the remaining pair remain current.

In the same way, from the algebraic mechanism

$$ax^m+by^m=c$$

we get the equation for the tangent line

$$ax_1{}^{m-1}x+by_1{}^{m-1}y=c.$$

Extending this thought to more complicated forms, we have the following divisions of algebraic mechanisms, using the word *simple* to indicate sets o terms containing only x's or only y's, and *compound* for terms containing both x's and y's.

A. Terms simple and composed of the same number of tracing points, that is of the same degree, as in the examples already given, $ax^2+by^2=c$. In these we adopt a *current pair* of coördinates and make all the others fixed, viz., $ax_1x+by_1y=c$.

Example. $x+3y=a$. To have a fixed point (point of tangency) we must have two mechanisms compounded, viz.,

$$x+3y+x+3y=2a,$$

whence, leaving one x and one y current, we get $x+3y+x_1+3y_1=2a$.

B. TERMS SIMPLE, BUT WITH AN UNEQUAL NUMBER OF TRACING POINTS, with the subdivisions as to variable terms.

B_1. TWO TERMS ONLY, example $y=x^2$.

B_2. MORE THAN TWO TERMS BUT EACH TERM BALANCED BY ITS MATE IN THE OTHER LETTER.

B_3. MORE THAN TWO TERMS BUT THE TERMS NOT BALANCED.

B_1. As in A, we make one pair of tracing points (coördinates) current, but here the potency of the tracing point is inversely proportional to the number required to constitute a term, and the number of terms from which a current y point must be taken to balance a current x point must be *directly* proportional to the number of points in the term (degree of the term). Thus in $y=x^3+b$, one y must be balanced by three x's. Hence, taking three mechanisms together, we get

$$y+2y=3x^3+3b.$$

Now having current one y and three x's, we get $y+2y_1=3xx_1^2+3b$.

Similarly in $y^2=x^3$, two y's must be balanced by three x's and we must take $2y^2+y^2=3x^3$.

Leaving current two y's and three x's, we get $2yy_1+y_1^2=3xx_1^2$.

Again in $y^2=4px$, one x balances two y's, hence $2y^2=4px+4px$. Leaving current one x and two y's we get $2yy_1=4px+4px_1$.

B_2. Here we put each set of homogeneous terms equal to a summand of the absolute term, operate on these equations by $A-B_1$, and add the results.

Example. $y+y^2=x+x^2+c$.

$$\begin{aligned} y-x&=p \qquad & y+y_1-(x+x_1)&=2p \\ y^2-x^2&=q \qquad & 2yy_1-2xx_1&=2q \end{aligned}$$

whence $y+y_1+2yy_1=x+x_1+2xx_1+2c$.

B_3. Here we take the sum of the y's (or x's) and break it up into the sum of term of degree similar to the original, each summand being put equal to a term of the x's (or y's), and apply $A-B_2$. Or take the sets of homogeneous terms and place them equal to summands of the constant term; operate on these summands by $A-B_2$ and add the results.

Example. $y+y^2=x$ $\qquad y=p \qquad y^2=q$

Whence $y+y_1=p+p_1$

$2yy_1=q+q_1$. Whence $y+y_1+2yy_1=x+x_1$.

Example. $y+y^2=x^3$. $\qquad y=p^3, \qquad y+2y_1=3pp_1^2$

$y^2=q^3, \qquad 2yy_1+y_1^2=3qq_1^2$

Whence $y+2y_1+2yy_1+y_1^2=3xx_1^2$.

Example. $y=x^3-x^2+x$. $\qquad p=x^3, \qquad p+p_1=3xx_1^2-p_1$

$q=-x^2, \qquad q+q_1=-2xx_1$

$r=x, \qquad r+r_1=x+x_1$

Whence $y+y_1=3xx_1^2-2xx_1+x+x_1-x_1^3$.

Example. $y^2=x^3-x^2$. $p^2=x^3$, $2pp_1+p_1^2=3xx_1$
$q^2=-x^2$, $2qq_1=-2xx_1$
$q_1^2=-x_1^2$

Whence $2yy_1+y_1^2=3xx_1^2-2xx_1-x_1^3$. Here q_1^2 is added so that we can sum up to y's.

Example. $y+y^2=x^3$. $y=p$, $y+y_1=2p$
$y^2=q$, $2yy_1=2q$
$-x^3=r$, $x_1^3-3xx_1^2=2r$

Whence $y+y_1+2yy_1+x_1^3-3xx_1^2=0$.

Example. $y+y^2=x^3$. $y=p$, $y+2y_1=3p$
$y^2=q$, $y_1^2+2yy_1=3q$
$-x^3=r$, $-3xx_1^2=3r$

Whence $y+2y_1+y_1^2+2yy_1-3xx_1^2=0$.

Example. $y=x^3-x^2$. $p=x^3$, $p+p_1=3xx_1^2-x_1^3$
$q=-x^2$, $q+q_1=-2xx_1$

Whence $y+y_1=3xx_1^2-x_1^3-2xx_1$.

Example. $y+y_2=-x^2$. $y=-p^2$, $y+y_1=-2pp_1$
$y^2=-q^2$, $2yy_1=-2qq_1$

Whence $y+y_1+2yy_1=-2xx_1$.

Example. $y+y^2=x^3-x^2+x$. $p+p^2=x^3$, $p+2p_1+2pp_1+p_1^2=3xx_1^2$
$q+q^2=-x^2$, $q+q_1+2qq_1=-2xx_1$
$r+r^2=x$, $r+2rr_1+r_1=x+x_1$

Whence $y+2yy_1+y_1=3xx_1^2-2xx_1+x+x_1-p_1-p_1^2=3xx_1^2-2xx_1+x-x_1^3+x_1^2$.

Example. $y+y^2=x^3-x^2+x$. $y-x=p$, $y+y_1-x-x_1=2p$
$y^2+x^2=q$, $2(yy_1+xx_1)=2q$
$-x^3=r$, $-3xx_1^2-x_1^3=2r$

Whence $y+y_1-x-x_1+2yy_1+2xx_1=3xx_1^2-x_1^3$.

The value for $2r$ is found as follows. Suppose r variable say z, then $z+2z=-3x^3$, $z+2z_1=-3xx_1^2$, but once z is really constant this becomes $2z_1=2r=-3xx_1^2-x_1^3$.

C. Terms compound, but of equal degree.

C_1. The x's and y's balanced.

C_2. The x's and y's not balanced.

C_1. As in A make one pair current, taking each element from a different term.

Example. $xy=1$. $xy+xy=2$, so that a current y may be balanced by a current x and leave a fixed point. Whence $xy_1+x_1y=2$.

Example. $x^2y^2=c$. $x^2y^2+x^2y^2=2c$. Whence $xx_1y_1^2+yy_1x_1^2=2c$.

Example. $x^3y^3=c$. $x^3y^3+x^3y^3=2c$. Whence $x_1^3y_1^2y+xx_1^2y_1^3=3c$.

C_2. As in B_3, break up the absolute term into summands, each summand being equal to one of the class of terms, and proceed as in B_1, making one pair current, selecting its constituents from a number of terms inversely proportional to the weight of the elements selected, operating upon the terms as classified in $A-C_1$.

Example. $x^2y^3=c$. Here two x's must be balanced by three y's. Hence $2xx_1y_1^3+3yy_1^2x_1^2=5c$.

Example. $x^2y+xy^2=a$. $\quad x^2y=a_1$, $\quad 2xx_1y_1+x_1^2y=3a_1$.
$\quad xy^2=a_2$, $\quad 2x_1y_1y+xy_1^2=3a_2$.

Whence $x(2x_1y_1+y_1^2)+y(2x_1y_1+x_1^2)=3a$.

D. TERMS COMPOUND AND NOT OF THE SAME DEGREE.

D_1. x'S AND y'S NOT BALANCED IN EACH TERM.

D_2. x'S AND y'S BALANCED IN EACH TERM.

D_1. Decompose into summands as in B_3, C_1 and add the results.

Example. $x^2y+x^2y^3=a$. $\quad x^2y=p$, $\quad x_1^2y+2x_1y_1x=3p$
$\quad x^2y^3=q$, $\quad 3x_1^2y_1^2+2x_1y_1^3x=5q$

Whence $x_1^2y+2x_1y_1x+3x_1^2y_1^2y+2x_1y_1^3x=3a+2q=3a+2x_1^2y_1^3$.

Example. $x^2y+x^3y=c$. $\quad x^2y=p$, $\quad x_1^2y+2xx_1y_1=3p$
$\quad x^3y=q$, $\quad x_1^3+3xx_1^2y_1=4q$

Whence $x_1^3y+3xx_1^2y_1+x_1^2y+2xx_1y_1=3c+x_1^3y_1$.

Example. $x^3y+xy=c$. $\quad x^3y=p$, $\quad x_1^3y+3xx_1^2y_1=4p$
$\quad xy=q$, $\quad x_1y+y_1x=2q$

Whence $x_1^3y+3xx_1^2y_1+x_1y+y_1x=2c+2p=2c+2x_1^3y_1$.

D_2. In D_1 owing to the unequal weight of x and y in the terms we are compelled by C_2 to take a number of terms proportional to the degree of the term before we can operate. In D_2 however, owing to the cancellations we must specifically designate this requirement or it will be overlooked. Hence, treat each term by itself and combine the results, each multiplied by the degree of its term.

Example. $xy+x^2y^2=a$. $\quad x_1y+xy_1+2(x_1^2y_1y+xx_1y_1^2)=2p+4q=2a+2x_1^2y_1^2$.

Example. $x^4y^4+xy=a$. $\quad x^4y^4=p$, $\quad 4(x_1^4y_1^3y+y_1^4x_1^3x)=8p$
$\quad xy=q$, $\quad x_1y+xy_1=2q$

Whence $4x_1^4y_1^3y+4xy_1^4x_1^3+x_1y+xy_1=8p+2q=2a+6x_1^4y_1^4$.

Example. $xy+x^3y^3=a$. $\quad xy=p$, $\quad x_1y+xy_1=2p$
$\quad x^3y^3=q$, $\quad 3(xx_1^2y_1^3+x_1^3y_1^2y)=3.2q$

Whence $x_1y+y_1x+3xx_1^2y_1^3+3x_1^3y_1^2y=2p+6q=2a+4x_1y_1+4x_1^3y_1^3$.

Example. $x^2y^2+x^3y^3=a$. $\quad 2(x_1xy_1^2+x_1^2yy_1)+3(xx_1^2y_1^3+x_1^3yy_1^2)=4p+6q=4a+2x_1^3y_1^3$.

E. COMPOSITES OF A, B, C, D.

Make each homogeneous set of terms equal to a summand of the absolute term. Operate on these by $A-D$, and add the results.

Example. $xy+x+y=a$. $\quad xy=p, \quad x_1y+xy_1=2p$

$x+y=q, \quad x+x_1+y+y_1=2q$

Whence $x_1y+xy_1+x+y+x_1+y_1=2x_1y_1+2x_1+2y_1$, and $x_1y+xy_1+x+y-x_1y_1=a$.

Example. $xy+x=1$. $\quad xy=p, \quad xy_1+x_1y=2p$

$x=q, \quad x+x_1=2q$

Whence $x_1y+xy_1+x+x_1=2(p+q)=2$.

Example. $x^2+y^2+xy=1$. $\quad x^2+y^2=p, \quad 2(xx_1+yy_1)=2p$

$xy=q, \quad xy_1+x_1y=2q$

Whence $xy_1+x_1y+2xx_1+2yy_1=2$.

Example. $x^2+xy=a$. $\quad x^2=p, \quad 2xx_1=2p$

$xy=q, \quad xy_1+x_1y=2q$

Whence $x_1y+xy_1+2xx_1=2a$.

PROOF THAT FOR MAXIMUM CURRENT THE EXTERNAL AND INTERNAL RESISTANCES SHOULD BE EQUAL.

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If we have a cells to connect we may take m series with n cells in each series. Then $mn=a$.

By formula for Ohm's law,

$$C=\frac{nE}{\frac{nr}{m}+R}$$

where r and R are respectively the internal resistance of each cell and the total external resistance.

Dividing numerator and denominator by n we have

$$C=\frac{E}{\frac{r}{m}+\frac{R}{n}}.$$

For maximum current it is necessary to make $\frac{r}{m}+\frac{R}{n}$ a minimum.

The expression takes the following form :